Sums of odd-ly many fractions and the distribution of primes

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In this talk, I will discuss new bounds on constrained sets of fractions. Specifically, I will discuss the answer to the following question, which arises in several areas of number theory: For an integer $k \geq 2$, consider the set of k-tuples of reduced fractions $\frac{a_1}{q_1}, \ldots, \frac{a_k}{q_k} \in I$, where I is an interval around 0. How many k-tuples are there with $\sum_i \frac{a_i}{q_i} \in \mathbb{Z}$?

When k is even, the answer is well-known: the main contribution to the number of solutions comes from "diagonal" terms, where the fractions $\frac{a_i}{q_i}$ cancel in pairs. When k is odd, the answer is much more mysterious! In ongoing work with Bloom, we prove a near-optimal upper bound on this problem when k is odd. I will also discuss applications of this problem to estimating moments of the distribution of primes in short intervals.