On non-coexistence of 2- & 3-rational caustics in nearly circular billiard tables

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A famous Birkhoff conjecture states that the only integrable convex planar billiards are billiards in an ellipse. We examined two closely related rigidity questions. A rational caustic is a caustic associated to a family of periodic orbits of the same period and the same rotation number. For example, a convex domain with a rational caustic of period two is a domain of a constant width. Elliptic billiard table admit rational caustic of any period greater than 2. Baryshnikov and Zharnitsky proved that an ellipse can be deformed so as to preserve any given rational caustic. The following question has been then proposed by Tabachnikov: are there nearly circular domains other than discs with two rational caustics of a prime period p and q? In this talk, I will discuss the following results:(rigidity) There are no nearly circular domains with two coexisting rational caustics of period two and three.(no super-rigidity) There may be infinitely many deformations of the circular domains with two coexisting rational caustics of period three and five with error given by the 3rd power of the perturbation parameter. Baryshnikov and Zharnitsky did prove that a properly chosen parametrization of the family D_n of billiard table with a rational caustic of period n give rise to a Hilbert sub-manifold of an appropriate Hilbert manifold. One can then wonder whether this manifold is a graph. Using a Nash-Moser-Zehnder generalized Implicit function Theorem, We showed that there exists an embedded continuous graph into D_m.This is based on a joint work with Vadim Kaloshin & Ke Zhang.

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