Random Cayley graphs

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We consider the random Cayley graph of a finite group G formed by picking k random generators uniformly at random:

(1) We prove universality of cutoff (for the random walk) and a concentration of measure phenomenon in the Abelian setup (namely, that all but $o(|G|)$ elements lie at distance $\in [R - o(R), R + o(R)]$ from the origin, where R is the minimal ball in $\mathbb{Z}^k$ of size at least $|G|$), provided $k \gg 1$ is large in terms of $\mathrm{rank}(G)$. As conjectured by Aldous and Diaconis, the cutoff time is independent of the algebraic structure (it is given by the time at which the entropy of a random walk on $\mathbb{Z}^k$ is $\log |G|$).

(2) We prove analogous results for the Heisenberg $H_{p,d}$ groups of $d \times d$ uni-upper triangular matrices with entries defined mod p, for p prime and d fixed or diverging slowly (as $p \to \infty$).

(3) Lastly, we resolve a conjecture of D. Wilson that if G is a group of size at most 2^d then for all k its mixing time in this model is as rapid as that of $\mathbb{Z}_2^d$ and likewise, that the slowest mixing p-group of a given size is $\mathbb{Z}_p^d$.

(Joint work with Sam Thomas.)

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